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Asymmetry identification in rigid rotating bodies—Theory and experiment

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ABSTRACT

Asymmetry and anisotropy are important parameters in rotating devices that can cause instability; indicate a manufacturing defect or a developing fault. The present paper discusses an identification method capable of detecting minute levels of asymmetry by exploiting the unique dynamics of parametric excitation caused by asymmetry and rotation. The detection relies on rigid body dynamics without resorting to nonlinear vibration analysis, and the natural dynamics of elastically supported systems is exploited in order to increase the sensitivity to asymmetry. It is possible to isolate asymmetry from other rotation-induced phenomena like unbalance. An asymmetry detection machine which was built in the laboratory demonstrates the method alongside theoretical analysis.

1. Introduction

Rotating elements e.g., gyroscopes [1], electrical motor cores [2] and aero-engine shafts [3] are never perfectly axisymmetric. Cyclic-symmetry in general is often part of the design and is not considered a fault nor is it a property the proposed method seeks to identify.

In some cases, asymmetry is undesired and can point to imperfections or faults in the rotating parts. Asymmetry can cause large vibrations due to parametric resonance [4] or point to undesirable changes in the rotating part, e.g. [5–7]. Asymmetry is normally not visible by sensors measuring radial vibrations, but is can still disrupt the mass balancing process which is essential for fast rotating machines [8].

The present paper, unlike prior publications, shows that stiffness changes in rotating and vibrating systems in general and nonlinear vibration analysis are not required for the purpose of asymmetry detection. Through rigid-body dynamics formulation and experiments it is demonstrated that rotation and asynchronous external excitation can be used isolate the level of asymmetry. The simplified analysis may assist in developing robust asymmetry detection devices, as is demonstrated here. The paper presents a non-dimensional measure of asymmetry which is directly related to measurable response amplitudes.

Indeed, it is well-known that any number cyclic divisions greater than two does not alter the isotropy of the inertia tensor around the axis of rotation. Asymmetry can alter the mass balancing processes and therefore affect the performance of a rotating device [7–9]. On the other hand, with two segments, e.g. two-blade propeller, the inertia tensor is no longer isotropic and it can bring about instability [4,10]. Stiffness asymmetry can be the result of a breathing crack in a rotating shaft [5,11]. Indeed a survey of such method aimed at detecting cracks is reported in [11,12] and [6,13] where some of these works are based on external active probing forces.

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The present paper seeks to identify mass asymmetry around the axis of rotation in rotating bodies by employing the fundamental equations of motion thus providing the relevant information. It does not seek to identify shaft-cracks in particular; although, it was shown [11,14,15] that it affects the nonlinear vibration and can be identified with relevant techniques. It was also shown that the same detection method can identify rotational mass asymmetry as well as stiffness asymmetry exhibited by cracks [16]. The present paper expands and simplifies the theory of asymmetry detection as well as illuminating some basic aspects of this approach.

The paper begins by analysis of the dynamics of a rigid rotating body with asymmetric inertia distribution under asynchronous excitation. Later, the analysis is expanded by considering an elastically supported device. The effect of larger asymmetry and unbalance are discussed. Finally, a laboratory experiment is shown and compared to the analytical results.

2. Increased sensitivity—exploiting parametric excitation

The basic physical phenomenon is demonstrated by an axisymmetric body on which 4 point-masses are glued to represented the added asymmetry [17,18], as illustrated in Fig. 1. The former model is a simplified model of a real device which is shown in Figs. 2 and 3.

The rigid body in Fig. 1 is rotating around two principle axes: (i) a constant speed rotation, Ω around $\mathbf{e}'_3, \mathbf{e}''_3$ and (ii) free oscillations $\dot{\phi}$ around $\mathbf{e}_1, \mathbf{e}''_1$. In the body-fixed coordinates, $\{\mathbf{e}'_1, \mathbf{e}'_2, \mathbf{e}'_3\}$, one can express the tensor of inertia by adding the contribution of the axisymmetric cylindrical body to the 4 point-masses (illustrated as spheres) whose location is determined by:

$$\mathbf{r}_1 = \rho\mathbf{e}'_1 + l\mathbf{e}'_3, \quad \mathrm{r}_2 = -\rho\mathbf{e}'_1 + l\mathbf{e}'_3, \quad \mathrm{r}_3 = \rho\mathbf{e}'_1 - l\mathbf{e}'_3, \quad \mathrm{r}_4 = -\rho\mathbf{e}'_1 - l\mathbf{e}'_3 \tag{1}$$

Adding the contribution of both elements, the axi-symmetric cylinder and the point masses, one obtains an expression for the tensor of inertia in body-fixed coordinates [18,19]:

$$\mathrm{I}'_{\boldsymbol{O}} = \mathrm{I}'_{\boldsymbol{O}} + \mathrm{I}'_{\boldsymbol{m}} = \begin{pmatrix} J_{\boldsymbol{d}}^0 & 0 & 0 \\ 0 & J_{\boldsymbol{d}}^0 & 0 \\ 0 & 0 & J_{\boldsymbol{p}}^0 \end{pmatrix} + \begin{pmatrix} 4ml^2 & 0 & 0 \\ 0 & 4ml^2 + 4m\rho^2 & 0 \\ 0 & 0 & 4m\rho^2 \end{pmatrix} \triangleq \begin{pmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_{\boldsymbol{p}} \end{pmatrix} \tag{2}$$

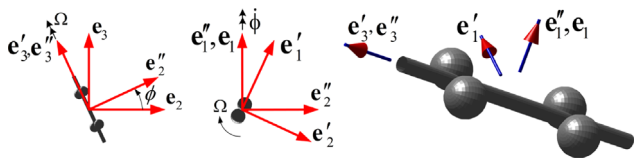

Fig. 1. A rigid rotating body with anisotropic inertia properties and frames of reference.

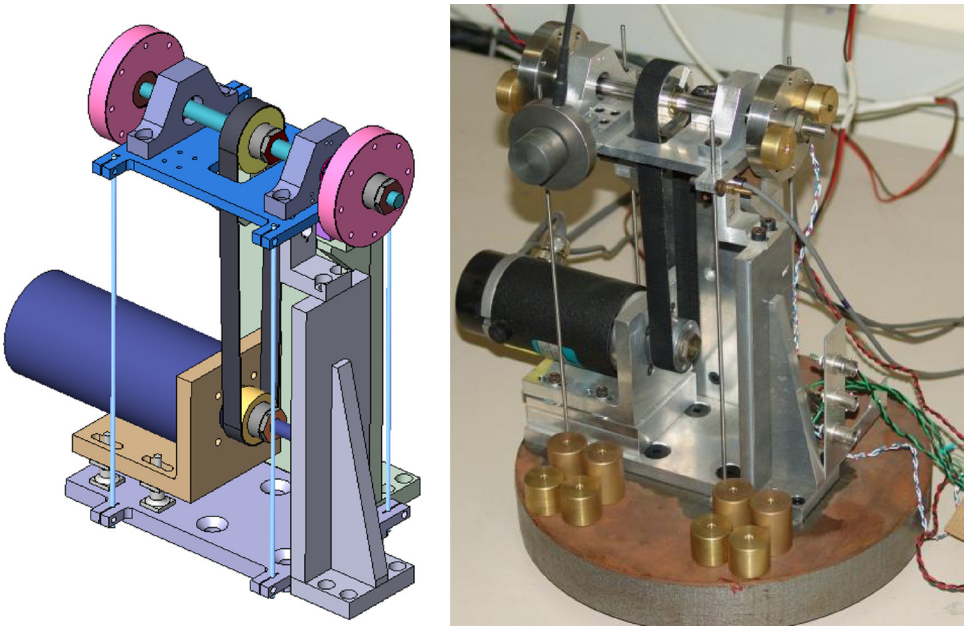

Fig. 2. Computer CAD drawing and a photograph of the asymmetry detection machine.

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The asymmetry is manifested in the tensor of inertia by the distance of the point masses, ρ, from the axis of spin, $\mathbf{e}'_3$. For notation and brevity purposes two parameters controlling the level of absolute and non-dimensional asymmetry are defined:

$$\Delta J \triangleq \frac{J_2 - J_1}{2}, \quad J_{\boldsymbol{avg}} \triangleq \frac{J_2 + J_1}{2}, \quad \alpha \triangleq \frac{\Delta J}{J_{\boldsymbol{avg}}} = \frac{J_2 - J_1}{J_2 + J_1} \tag{3}$$

Clearly, ΔJ, α are both zero for perfectly axi-symmetric bodies and $|\alpha| \leq 1$ otherwise.

Consider the oscillations of the system in the $\{\mathbf{e}''_1, \mathbf{e}''_2, \mathbf{e}''_3\}$ frame of reference. Due to rotation, the tensor of inertia in this frame of reference becomes time-dependent:

$$\mathbf{I}''_0 = \begin{pmatrix} J_{avg} & 0 & 0 \\ 0 & J_{avg} & 0 \\ 0 & 0 & J_p \end{pmatrix} - \Delta J \begin{pmatrix} \cos 2\Omega t & \sin 2\Omega t & 0 \\ \sin 2\Omega t & -\cos 2\Omega t & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{4}$$

As can be seen in Eq. (4), the time-dependent part is proportional to the level of asymmetry.

Now, the level of asymmetry is assumed to be small and therefore hard to detect or isolate from the effect of unbalance at the speed of rotation. For this purpose, an external, non-synchronous excitation is introduced and the equation of motion is derived for this very case. The balance of angular momentum around the center of mass under the influence of an external moment, yields [19]:

$$\mathbf{T}_{\boldsymbol{o}} = \frac{\boldsymbol{d}}{\boldsymbol{d}t} \mathrm{h}_{\boldsymbol{G}} = \frac{\boldsymbol{d}}{\boldsymbol{d}t} (\mathrm{I}''_0 \boldsymbol{\lambda}) + \dot{\phi} \mathbf{e}''_1 \times (\mathrm{I}''_0 \boldsymbol{\lambda}) \tag{5}$$

Where the sinusoidal external moment and the angular velocity vector are defined as:

$$\mathbf{T}_{\boldsymbol{O}} = Q_{\boldsymbol{e}} \sin \omega t \, \mathbf{e}''_1, \quad \boldsymbol{\lambda} = \dot{\phi} \mathbf{e}''_1 + \Omega \mathbf{e}''_3 \tag{6}$$

The equation of motion around $\mathbf{e}''_1$, $\mathbf{e}_1$ becomes:

$$\frac{\boldsymbol{d}}{\boldsymbol{d}t} \left\{ (J_{\boldsymbol{avg}} + \Delta J \cos 2\Omega t) \dot{\phi} \right\} + 2 \Delta J \Omega \sin 2\Omega t \dot{\phi} = Q_{\boldsymbol{e}} \sin \omega t \tag{7}$$

The effect of asymmetry is to modulate the inertia coefficients and therefore it makes the equation of motion time-dependent. A solution for the steady-state response of Eq. (7) can be sought in the form of an infinite series according to [20] using a truncated approximation in a similar manner to [21]. Substituting,

$$\dot{\phi} \approx \sum_{n=0}^{N} a_n \cos(\omega + n2\Omega)t + b_n \sin(\omega + n2\Omega)t \tag{8}$$

and balancing the different frequencies appearing in Eq. (8) one can obtain a set of $2(N+1)$ linear equations, leading to:

$$\dot{\phi} \approx -\frac{Q_{\boldsymbol{e}}}{2 J_{\boldsymbol{avg}} \omega} \left(\frac{\Delta J}{J_{\boldsymbol{avg}}} \right) (\cos(2\Omega + \omega)t + \cos(2\Omega - \omega)t) - \frac{Q_{\boldsymbol{e}}}{J_{\boldsymbol{avg}} \omega} \cos \omega t + O\left(\left| \frac{\Delta J}{J_{\boldsymbol{avg}}} \right|^2 \right) \tag{9}$$

The effect of choosing a higher order approximation given larger asymmetry is discussed in the appendix. In the former expression it was assumed that asymmetry is small and therefore higher order terms were neglected.

It is clear that the effect of asymmetry is to create additional frequency terms whose magnitude is proportional to the level of asymmetry previously defined as $\alpha \triangleq (\Delta J / J_{\boldsymbol{avg}})$. Furthermore, only the asymmetry related terms are modulated by rotation speed and external excitation and are therefore represented here by the isolated frequencies, $2\Omega \pm \omega$.

At tuned external excitation has a crucial role in the detection of asymmetry. Without excitation, asymmetry has zero effect on the measured response. The frequency of external excitation can be chosen at will so as to isolate the asymmetry related spectral line from other frequencies that may occur naturally in rotating structures and thus provide ambiguous results.

Although asynchronous external excitation isolates the effect of asymmetry, it is clear from Eq. (9) that these terms can be rather small. The small asymmetry parameter is divided by the inertia term and therefore it could be difficult to detect under normal signal to noise conditions at reasonably high frequencies. It turns out that elastically supported devices can be driven by tuned external excitation frequency that amplifies the effect of asymmetry to a level it can be easily detected (see also [16]).

Consider the device shown in Fig. 2 which contains the potentially asymmetric rotating part of Fig. 1. The rotating part is mounted on a rigid platform that allows motions restricted to a plane parallel to the base. The long rods in Fig. 2 are rigid longitudinally and flexible in bending thus allowing only the aforementioned type of motions. The shaft is rotated by an electric motor coupled to the shaft by a flexible belt.

The elastic rods have negligible mass and their effect in rotation around the mass center adds a stiffness term to Eq. (7) thus the same motions are now governed by:

$$\frac{d}{dt} \left\{ (J_{\boldsymbol{avg}} + \Delta J \cos 2\Omega t) \dot{\phi} \right\} + 2 \Delta J \Omega \dot{j} \sin 2\Omega t + k_\phi \phi = Q_{\boldsymbol{e}} \sin \omega t \tag{10}$$

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here, k_ϕ represents the rotational stiffness around $\mathbf{e}_1$. Eq. (10) considers only the rotation around $\mathbf{e}_1$ (apart from the spin) and it can be solved for the steady-state response by proposing a series solution similar to Eq. (8). Performing the harmonic balancing process and keeping first order terms of the asymmetry coefficient, one obtains:

$$\phi \approx -\frac{Q_{\boldsymbol{e}}}{2J_{\boldsymbol{avg}}(\omega^2-\omega_{\boldsymbol{n}}^2)}\left(\frac{\Delta J}{J_{\boldsymbol{avg}}}\right)\left(\frac{(4\Omega+\omega)\omega}{(\omega-\omega_{\boldsymbol{n}}+2\Omega)(\omega+\omega_{\boldsymbol{n}}+2\Omega)}\sin(2\Omega+\omega)t-\right.$$
$$\left.-\frac{(4\Omega-\omega)\omega}{(\omega-\omega_{\boldsymbol{n}}-2\Omega)(\omega+\omega_{\boldsymbol{n}}-2\Omega)}\sin(2\Omega-\omega)t\right)-\frac{Q_{\boldsymbol{e}}}{J_{\boldsymbol{avg}}(\omega^2-\omega_{\boldsymbol{n}}^2)}\sin\ \omega t+O(\alpha^2) \tag{11}$$

where the definition $\omega_{\boldsymbol{n}}^2 \triangleq (k_\phi/J_{\boldsymbol{avg}})$represents the average non-rotating natural frequency.

It is worth reminding that the governing equation, i.e. Eq. (10) is linear with periodically time varying coefficients. The additional frequencies appear because of rotation and asymmetric related modulation of the coefficients and are not related to nonlinear effects. Due to linearity, different types of excitation forces can be treated independently, e.g. response to unbalance.

Examination of Eq. (11) reveals that specific choices of excitation frequency can increase the asymmetry related terms considerably. In reality, the amplitude of these terms will be limited by damping and possibly nonlinear and saturation effects that are not considered here. There are 6 obvious choices that would increase the effect of asymmetry very much, these are:

$$\omega = \pm\,\omega_{\boldsymbol{n}}, \quad \omega = \pm\,\omega_{\boldsymbol{n}} \pm 2\Omega \tag{12}$$

The first two frequencies give rise to response at the frequency of excitation and therefore cannot be fully associated with asymmetry. The other 4 options create a parametric excitation completely separate from all other sources. The latter can be controlled by changing both the speed of rotation and the external excitation until near resonance condition appears. In practice, since asymmetry is small, the response amplitude will be limited but still large enough for detection purposes. This fact will be demonstrated later in an experiment. It is worth mentioning that a poor choice of excitation frequency, i.e.,

$$\omega = \pm\,\frac{\Omega}{4} \tag{13}$$

can nullify some of the spectral lines associated with unbalance. Indeed, some former studies [22] have shown that certain spectral lines disappear at certain speeds of rotation which could be a detection and verification means in some cases.

2.1. The effect of unbalance in presence of asymmetry

Rotating devices always contain some degree of imperfections that separates their axis of rotation from the center of mass or from the principal axis of inertia [19]. Considering the asymmetry detection device in Fig. 2, one can consider the effect of unbalance by substituting in Eq. (11):

$$\omega = \Omega, \quad Q_{\boldsymbol{e}} = m_{\boldsymbol{u}}\varepsilon_{\boldsymbol{e}}\,\Omega^2 \tag{14}$$

Here, $m_{\boldsymbol{u}}\varepsilon_{\boldsymbol{e}}$ defines the amount of unbalance [21].

The steady-state response in this case, due to the combined effect of small asymmetry and unbalance is:

$$\phi \approx -\frac{m_{\boldsymbol{u}}\varepsilon_{\boldsymbol{e}}\Omega^4}{2J_{\boldsymbol{avg}}(\Omega^2-\omega_{\boldsymbol{n}}^2)}\left(\frac{\Delta J}{J_{\boldsymbol{avg}}}\right)\left(\frac{5}{(3\Omega-\omega_{\boldsymbol{n}})(\omega_{\boldsymbol{n}}+3\Omega)}\sin\ 3\Omega t+\frac{3}{(\omega_{\boldsymbol{n}}+\Omega)(\omega_{\boldsymbol{n}}-\Omega)}\sin\ \Omega t\right)-\frac{m_{\boldsymbol{u}}\varepsilon_{\boldsymbol{e}}\Omega^4}{J_{\boldsymbol{avg}}(\Omega^2-\omega_{\boldsymbol{n}}^2)}\sin\ \omega t \tag{15}$$

Superficially, it seems that ordinary unbalance excitation can be useful in detecting asymmetry. Indeed, it has been reported in the past e.g. [12,22,23] that the asymmetry caused by cracked shafts responds to unbalance by creating integer multiples of rotation speed. But for the purpose of detection, the latter option is less appealing because harmonics of rotation speed can be caused by several sources, e.g. nonlinearity and static run-out [24]. As was discussed before, external, asynchronous excitation seems advantageous in this respect.

It is shown analytically in the appendix and in the experimental study below that the number of excited harmonics is greater than what appears in truncated approximation given in Eq. (15). It is also clear that higher terms and higher sidebands have diminishing magnitudes.

3. Experimental study

A laboratory asymmetry detection system was designed and built as shown in Figs. 2 and 3. The system includes an external excitation device (see Fig. 3) and a deliberate, controllable asymmetry in the form of 4 added Brass masses that are visible in the photograph.

Several cases were analyzed in the experimental study in an attept to isolate the effects of unbalance, asymmetry, asynchrounous excitation and inherent dynamical effect attributed to the mechanical system.

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In the first run the system, as shown in Fig. 2 without the induced asymmetry, was run at constant speed. The measured response was processed and displayed in Figs. 4–6. Fig. 4 presents the spectra for the unbalanced and balanced cases. The amplitude at speed of rotation was descreased dramatically following a mass-balancing operation where the effect of unabalnced was minimized [24]. The effect of the belt driving mechanism, which has some dynamics of its own, appears at low frequency and it modulates the speed of rotation to exhibit a spectral line at 56 Hz.

The effect of the driving-belt dynamics can be eliminated by means of time-domain averaging (TDA) [25] which exploits the once per revolution pulse provided by an optical sensor (trapezoidal shaped sensor in Fig. 3). The effect of TDA is to eliminate the non-periodic components leaving only integer multiples of rotation frequency as can be seen in Fig. 5.

In order to assess the effect of asymmetry, given the residual unbalance excitation, the system was run in two configurations: symmetric (without the added Brass masses) and asymmetric. Where the Brass masses shown in Fig. 3 were connected. In logarithmic scale, the two cases exhibit multiple harmonics (those are nearly invisible in linear scale). There is a noticeable different (see Fig. 6) in the asymmetric case, but without baseline reference data, it is impossible to indicate which data belongs to an asymmetric device.

Finally, the proposed method was attempted whereby an asynchronous external excitation was applied. The two cases exhibit visibly-similar time-domain responses, as shown in Fig. 7.

Clearly, the ability to isolate the effect of asymmetry, as manifested by Eq. (11), should be examined in the frequency domain. A comparison of a symmetric and asymmetric systems is shown in Fig. 8.

Evidently, once the external excitation was applied at a frequency of 413 Hz, two sidebands appeared in asymmetric case at $413 \pm 2 \times 42.67$ Hz, and $413 \pm 4 \times 42.67$ Hz while no effect was notices in the symmetric one. The effectiveness of the proposed approach was clearly demonstrated in an experiment.

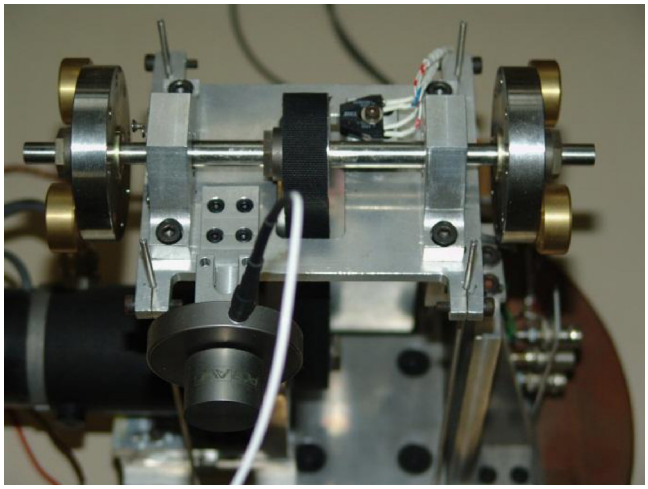

Fig. 3. Asymmetry detection machine, top view showing the external proof-mass actuator and the induced asymmetry realized by added Brass masses.

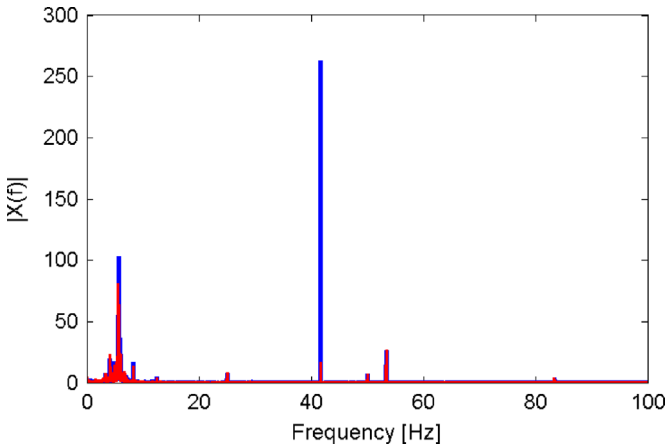

Fig. 4. Spectra at constant speed for: ——— unbalanced and ——— balanced cases.

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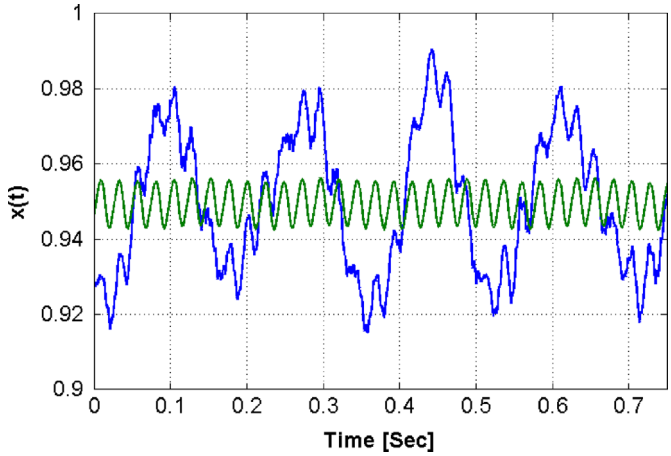

Fig. 5. Time domain measurement: raw data and: averaged data after 1000 TDA rotation-synchronous averages.

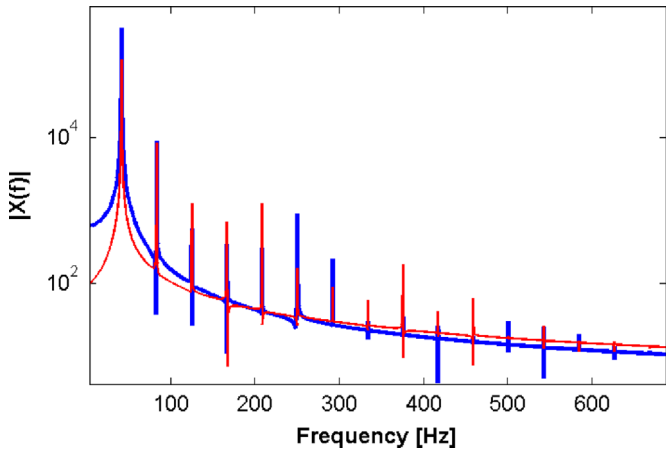

Fig. 6. Spectra of measured response at constant speed of symmetric and asymmetric rotors after TDA was performed.

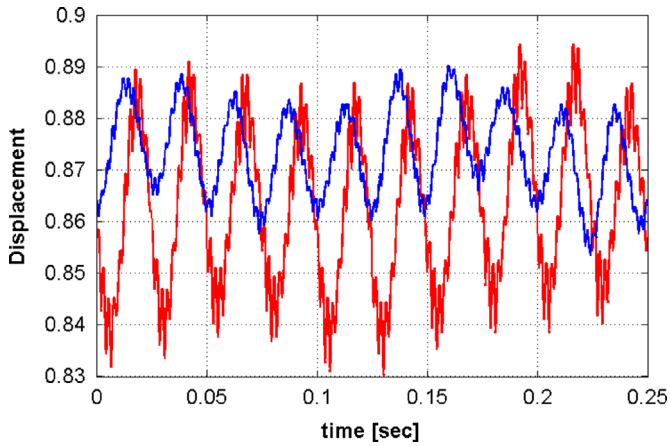

Fig. 7. Time domain measurement (raw): asymmetric system and nominally rotationally-symmetric rotor.

4. Conclusions

Asymmetry is present in any real-world device and it can be important in some cases even when it is small. Asymmetry can affect the performance of gyroscopes, electrical motors or be an indication of a developing mechanical fault. Although only inertia-related asymmetry is studied here, the same approach can be used to detect stiffness asymmetry (i.e. cracked

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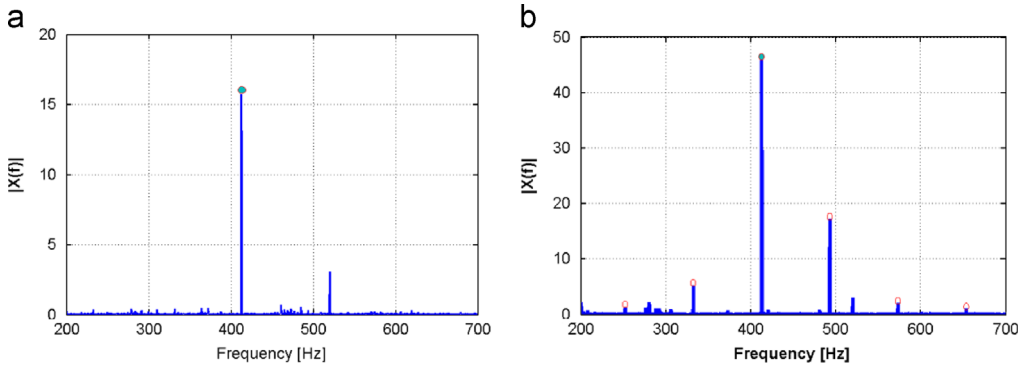

Fig. 8. Spectra of measured signals. (a) symmetric rotor. (b) asymmetric rotor. Indicative side-bands appear at $413 \pm 2n42.67$ Hz in the asymmetric configuration and vanish in the symmetric set-up. Filled circle indicate the asynchronous excitation frequency, hollow circles indicate sidebands (Linear scale).

shafts) or uneven distribution of magnetic forces. The proposed approach employs active probing by introducing an external, asynchronous sinusoidal excitation. Once the frequency of this probing force is tune to generate a suitable combination with speed of rotation, high sensitivity to asymmetry can be achieved. The method can be viewed as a tuned parametric excitation that harnesses the natural compliance of elastic structures to amplify an otherwise small effect.

The method was analyzed using analytical tools and it was shown that it can be explained using a linear, although time-dependent, differential equation. Rotation combined with external excitation provides richer information about asymmetry than non-rotating systems. An asymmetry detection machine was built and demonstrated in this work. Some signal processing of the real-worked data was presented and good agreement with theory was obtained.

Acknowledgments

The first author would like to acknowledge the contribution of the late Dr. Y. Porat for suggesting the topic and for the discussions and design of the test-rig through his vast knowledge in dynamics and engineering. This paper is dedicated in memory of Dr. Porat, my graduate-school dynamics teacher. The authors are grateful to Mr. Hadar Raz for realizing the test rig so skillfully.

Appendix A. Rigid asymmetric rotor—derivation

Consider a steady-state solution for Eq. (7) using a truncated series where:

$$\boldsymbol{\theta} \triangleq \begin{pmatrix} 4\Omega+\omega & 2\Omega+\omega & \omega & 2\Omega-\omega & 4\Omega-\omega \end{pmatrix} \tag{16}$$

is the set of frequencies considered in the steady-state solution. Accordingly, define two 5×1 vectors of coefficients whose entries $(a_n,\ b_n)$, $n = 1\ldots5$, multiply the cosine/sine terms:

$$a_n \cos\theta_n t, \quad b_n \sin\theta_n t, \quad n = 1\ldots5 \tag{17}$$

The harmonic balancing process equates the terms in Eq. (17) to produce a set linear of equations:

$$\begin{pmatrix} 0 & -\mathbf{A}_0 \\ \mathbf{A}_0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix} = \mathbf{r} \tag{18}$$

Where the definitions in Eq. (3) was used to write:

$$\mathbf{A}_0 \Omega \begin{pmatrix} -4 & 2\alpha & 0 & 0 & 0 \\ \alpha & -2 & \alpha & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\alpha & -2 & \alpha \\ 0 & 0 & 0 & 2\alpha & -4 \end{pmatrix} + \frac{\omega}{2} \begin{pmatrix} -2 & \alpha & 0 & 0 & 0 \\ \alpha & -2 & \alpha & 0 & 0 \\ 0 & \alpha & -2 & -\alpha & 0 \\ 0 & 0 & \alpha & 2 & -\alpha \\ 0 & 0 & 0 & -\alpha & 2 \end{pmatrix} \tag{19}$$

The solution of Eq. (18) yields Eq. (9). Alternatively, considering a solution up to the order α^4, additional frequency terms appear and all the amplitudes are slightly modified.

$$\dot{\phi} \approx -\frac{Q_e}{2J_{avg}\omega}\alpha\left(\tfrac{3}{4}\alpha^2+1\right)\left(\cos(2\Omega+\omega)t + \cos(2\Omega-\omega)t\right) +$$

$$\frac{Q_e}{4J_{avg}\omega}\alpha^2\left(\frac{3}{4}\alpha^2+1\right)(\cos(4\Omega+\omega)t+\cos(4\Omega-\omega)t)+\frac{Q_e}{4J_{avg}\omega}(4+2\alpha^2+3\alpha^4)\cos\omega t+O(\alpha^5) \tag{20}$$

Appendix B. Rigid asymmetric rotor on elastic supports

A higher order approximation for the steady-state solution of Eq. (10) can be obtained in a nearly identical manner to the derivation in Appendix A. It turns out that a second order approximation in the asymmetry parameter modifies only 3 frequency terms. The added terms due to the higher order approximation need to be combined with Eq. (11) are:

$$\Delta\phi\approx\Delta b_1\cos(4\Omega+\omega)t+\Delta b_3\cos\omega t+\Delta b_5\cos(4\Omega-\omega)t \tag{21}$$

where:

$$\Delta b_1=-\frac{Q_e\alpha^2}{4J_{avg}(\omega^2-\omega_n^2)}\frac{\omega\,(2\,\Omega+\omega)\,(4\,\Omega+\omega)\,(6\,\Omega+\omega)}{(\omega-\omega_n-2\Omega)(\omega+\omega_n-2\Omega)(\omega-\omega_n+2\Omega)(\omega+\omega_n+2\Omega)}$$

$$\Delta b_3=\frac{Q_e\alpha^2}{2J_{avg}(\omega^2-\omega_n^2)}\frac{\omega^2\,(4\,\Omega^2+\omega^2)\,(12\Omega^2-\omega^2+\omega_n^2)}{(\omega-\omega_n-2\Omega)(\omega+\omega_n-2\Omega)(\omega-\omega_n+2\Omega)(\omega+\omega_n+2\Omega)}$$

$$\Delta b_5=-\frac{Q_e\alpha^2}{4J_{avg}(\omega^2-\omega_n^2)}\frac{\omega\,(2\,\Omega+\omega)\,(4\,\Omega-\omega)+(6\,\Omega-\omega))}{(\omega-\omega_n-2\Omega)(\omega+\omega_n-2\Omega)(\omega-\omega_n+2\Omega)(\omega+\omega_n+2\Omega)}$$

Considering higher orders in the approximation, additional harmonics appear. Indeed, it can be assumed that for larger asymmetry, higher sideband harmonics will become larger. There is also some effect on the amplitude at the frequency of excitation as indicated by$-\Delta b_3$. Finally, the sidebands disappear at some sub-harmonics where the numerators of these coefficients become zero.

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