Experimental Identification of Nonlinearities under Free and Forced Vibration using the Hilbert Transform

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Abstract: In this paper we discuss the experimental identification of a nonlinear vibrating mechanical system. The system under test incorporated several spring and damping related nonlinearities. Indeed, in this paper we use data from a real laboratory device thus increasing the confidence in the proposed methods that have been previously applied mostly to simulated data. A unique feature of the identified model is that it shows the dependency of the estimated parameters on the vibration amplitude. The provided measurements of free and forced vibration motion, together with the unique signal processing, based on the Hilbert transform analysis, yield an accurate estimation of nonlinear spring and friction parameters of the vibration model. The obtained natural frequencies and friction parameters are functions rather than scalars that describe the system's behavior under different operating conditions. This paper complements previously published Hilbert transform analytical methods with experimental and numerical results.

Keywords: Nonlinear dynamic system identification, Hilbert transform.

1. INTRODUCTION

Nonparametric identification of nonlinear vibrating oscillators deals with the construction of initially unknown functions of nonlinear restoring forces and damping. This problem, which is categorized as an inverse problem, can be difficult to solve since it is can be difficult to find suitable parametric models for such systems. Furthermore, many parameterization schemes can provide nonunique solutions that lack a physical interpretation. The described approach circumvents many of the difficulties by virtue of it being nonparametric and thus reflects the actually measured characteristics faithfully.

Typically, every nonlinear equation, describing vibration motion, has a fixed structure. The structure classically includes three independent members: a restoring elastic force (stiff-

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Figures 1–7 appear in color online: http://jvc.sagepub.com

ness, spring), which is a nonlinear function of displacement (position), a damping force (friction), which is a nonlinear function of velocity (the first derivative of position with respect to time), and an inertial force proportional to acceleration (the second derivative of position with respect to time). Every independent restoring and damping force member is an unknown nonlinear function of motion. For example, a hardening or softening spring whose stiffness depends on the displacement, friction or damping parameter that depends on the state (velocity) rather than being a fixed, constant scalar parameter.

By observation (experiment), one acquires knowledge of the position and/or velocity of the vibrating object as well as the excitation at several known instants of time. The proposed nonparametric identification determines the initial, nonlinear restoring and damping forces. In the case of free vibration one can acquire only the response signal, the vibration of the oscillators, whereas in the case of forced vibration one can utilize both the measured excitation and response.

Over the last two decades a considerable number of published papers have been devoted to Hilbert transform (HT)-based identification and specifically to nonlinear vibrating systems (Worden and Tomlinson, 2001; Huang and Shen, 2005; Kerschen et al., 2006). Recently, the HT identification method has found wider application alongside with the modern empirical mode decomposition method for the analysis and identification of dynamic structures.

A nonlinear vibrating structure consisting of several components can exhibit two different physical natures of vibration motions simultaneously. The first is just a multi-component oscillation induced by partial motion of each of the coupled subsystems. These coupled vibration components also exist in linear multiple-degree-of-freedom (MDOF) systems, but nonlinearity can cause energy exchange at different frequencies between the coupled subsystems. The second type is associated with the intricate nonlinear relationships in restoring and damping forces, which produce superharmonics and inter-modulation. These superharmonics appear together with the primary (principle) solution only in nonlinear system.

The current research literature presents various techniques for the identification and analysis of MDOF nonlinear oscillators. Modern HT-based decomposition methods separate nonlinear oscillations into uncoupled subsystem coordinates for identification of every nonlinear normal mode as a simple single-degree-of-freedom (SDOF) system (Pai, 2007; Kerschen et al., 2008). In practice, when a weakly nonlinear system's response is mostly dominated by the main primary nonlinear solution and when the coupling strength between the coupled subsystems is small, we can consider only a single principal component of the main mode of the solution.

In the majority of the researched cases, a spatial nonlinear parametric model is required for proper system identification. For example, when studying the nonlinear equations of motion, Poon and Chang (2007) used an approximate solution for the normal mode invariant manifold near the equilibrium point in the form of a third-order Taylor series expansion. In principle, when a detailed nonlinear model structure and the nature of the solution are known, the identification can be converted into a parametric identification problem, where unknown parameters can be derived just by fitting an algebraic expression to the data. These parametric identification techniques, based on the *a priori* nonlinear model, are only suitable for the specific chosen model. More preferable are nonparametric identification methods which do not require an *a priori* parameterization of a nonlinear model. Such methods should detect and characterize the type and the degree of nonlinearity, the structure of the unknown vibration model and the parameters of this model, while considering only measured vibration and excitation data. The HT nonparametric identification methods identify physically meaningful parameters of the spring–damper elements of real mechanical structures.

Some research works focusing on the HT identification method extract a typical nonlinear model, but only with a single nonlinear element whether it is a backlash (Tjahjowidodo et al., 2007) or a structural viscous damping (Ereta and Meskellb, 2008).

The goal of the current work is to present an experimental nonparametric HT identification of a real nonlinear vibration system with several common nonlinear spring and damping elements that appear concurrently. The identified system is required to exhibit a dominant primary solution under free and forced vibration.

Some typical mechanical systems having elastic and damping nonlinearities are reviewed in Sections 2 and 3: these typically arise at the small or large vibration amplitude range only. We describe the HT algorithms for estimating the instantaneous modal parameters in Section 4. In Section 5 the tested mechanical structure under free and forced vibration regimes is discussed. We discuss the main contribution of this paper in Section 6, where experimental results in the form of the identified nonlinear equations are produced.

2. ELASTIC NONLINEARITIES IN VIBRATION SYSTEMS

This important type of nonlinearity arises when the restoring force of a spring is not proportional to its deformation. There are several known types of static force characteristic (load–displacement curve) representing different types of nonlinearity in elastic springs: backlash, preloaded (pre-compressed), impact and polynomial. A vibrating system, normally described by fixed parameters, can be described by a piecewise-linear restoring force that may be considered as an approximation to continuous typical curves (Feldman and Braun, 1993). Vibrating systems can have symmetric static force–displacement characteristics (symmetric with respect to the origin) as well as nonsymmetric characteristics of the nonlinear restoring force.

In most nonlinear vibration systems the natural frequency will be decisively dependent on the amplitude of the vibrations. Therefore, typical nonlinearities in springs have a unique form of skeleton (backbone) curve (Feldman and Braun, 1993). The topography of the skeleton curve, which is unique, is essential in assessing the properties of the tested vibrating system, e.g. in reconstructing the characteristics of the nonlinear elastic forces. Let us consider two relevant cases of nonlinear elastic forces in a SDOF vibrating system.

2.1. A Preloaded Model (Small-amplitude Nonlinear Behavior)

There are cases where vibrating systems show their specific nonlinear behavior only in the small-amplitude range of vibrations. One such system is a spring having backlash (clearance). For large amplitudes, the backlash nonlinearity is negligible and a linear model yielding constant natural frequency satisfies the requested accuracy. However, for small amplitudes of vibration, the system will display its nonlinear properties where its natural frequency decreases with the decrease in amplitude.

Another typical example of nonlinearity in the small-amplitude range is a vibrating mechanical system with preloading (pre-tension). Actually, for large vibration amplitudes the

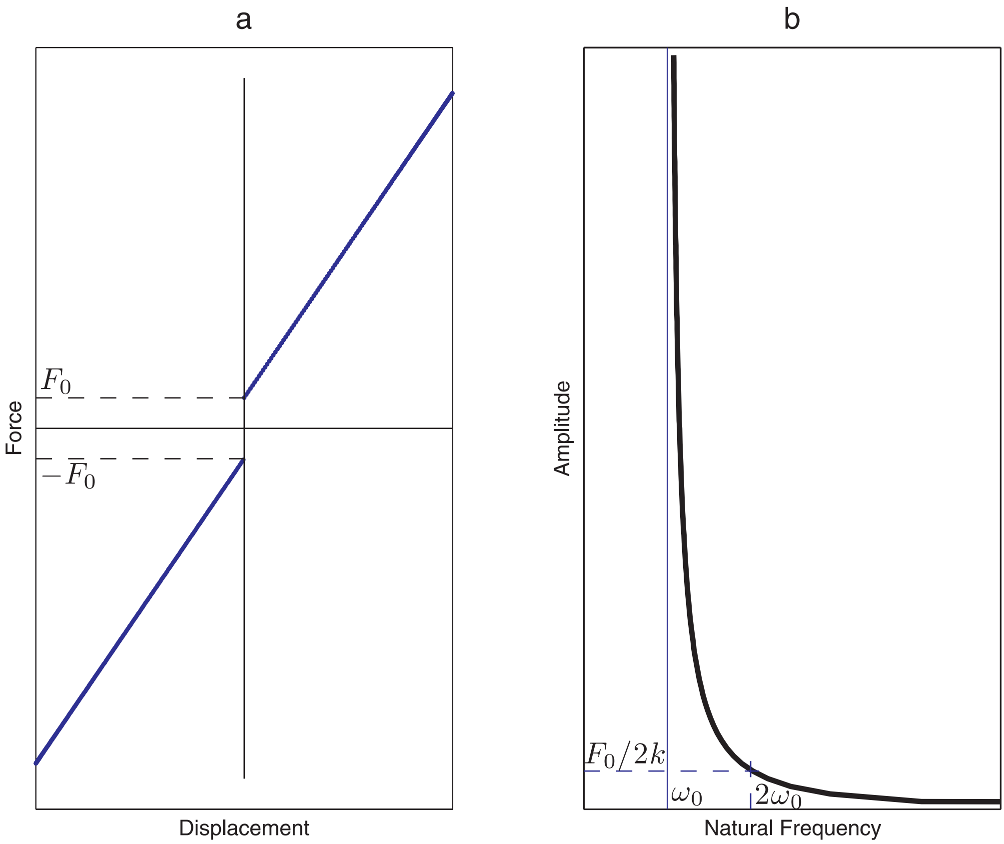


Figure 1. The preloaded stiffness model: (a) the static force characteristics; (b) the skeleton curve.

natural frequency, in this case, does not depend on the vibration amplitude. The natural frequency will only increase at an extreme rate for small amplitudes of oscillatory motion commensurable to a preloaded deformation. Let us consider a model having a stiffness k, in which springs are preloaded (pre-tensioned) by a force having a magnitude F_0, as indicated by the ideal static load–displacement diagram in Figure 1a. For this preloaded model, an expression for the natural undamped frequency takes the form (Feldman and Braun, 1993):

$$\omega(A) = \frac{\pi \omega_0}{2\arccos \dfrac{1}{1 + Ak/F_0}}. \tag{1}$$

The proper backbone curve of the system is located to the right of the asymptote ω_0 when $F_0 = 0$ and the natural frequency grows towards infinity for smaller and smaller amplitudes of vibration owing to the presence of the preload nonlinearity (Figure 1b). At low amplitudes, the value of the natural undamped frequency is doubled ($2\omega_0$) at an amplitude that is half of F_0/k (Figure 1). The resulting equation of free vibrations, not considering the effect of damping, will take the following form:

$$m\ddot{x} + F_0 \operatorname{sgn}(x) + kx = 0; \quad \ddot{x} + \frac{F_0}{m} \operatorname{sgn}(x) + \omega_0^2 x = 0. \tag{2}$$

The second, normalized form is commonly used in the identification.

2.2. Polynomial Model (Large-amplitude Nonlinear Behavior)

Most of the known cases of nonlinear manifestation occur in the large-amplitude range of oscillation. Typical examples consist of a nonlinear spring element with hardening or softening restoring force, and nonlinear friction that is a quadratic or cubic force. For large amplitudes of vibration the occurrence of these spring or damping nonlinearities cannot be ignored. Typical nonlinear spring elements of a mechanical vibration system could be represented as a power series

$$k(x) = (\alpha_1 + \alpha_3 x^2 + \alpha_5 x^4 + \cdots)x. \tag{3}$$

In particular, a system described by a Duffing equation has only a cubic spring member and its backbone skews towards higher frequencies in the case of a hardening (the positive cubic member) spring. In general, a second-order conservative system with a nonlinear restoring force $k(x)$ and a solution $x(t) = A\cos\omega t$ takes a simple form

$$\ddot{x} + k(x) = 0. \tag{4}$$

Applying the multiplication property of the HT for overlapping functions (Feldman, 1997) to Equation (4), we obtain a new form of time-varying equation of motion

$$\ddot{x} + j\delta(t)x + \omega_0^2(t)x = 0, \tag{5}$$

Here $\omega_0(t)$ is the fast varying natural frequency function. A new term $\delta(t)$, which is a fictitious hysteretic damping-like coefficient, appears due to the transformation. This term has no effect on the average energy per cycle and therefore it does not affect the average natural frequency (Feldman, 1997).

If we consider only the mean value of the varying squared natural frequency function, we obtain an important result

$$\left\langle \omega_0^2 \right\rangle = T^{-1} \int_0^T \omega_0^2(t)dt = \alpha_1 + \frac{3}{4}\alpha_3 A^2 + \frac{5}{8}\alpha_5 A^4 + \cdots, \tag{6}$$

which proves that the averaged natural frequency $\left\langle \omega_0^2 \right\rangle$ is similar to the initial nonlinear restoring force $k(x)$ (Equation (3)) with slight difference in the polynomial coefficients. This general result means that the estimated average natural frequency and hence the system skeleton curve (backbone) $A(\langle\omega_0\rangle)$ include the main information about the initial nonlinear elastics characteristics and can be used for nonlinear system identification.

2.3. Combination of Different Elastic Elements

A real vibrating system could have several nonlinear, parallel-acting springs, whose equivalent spring force is the sum of the forces of the individual models, when all of the springs have the same deformation. In general, the task of decomposing the obtained backbone as a sum of typical curves has no simple solution. Nonetheless, in the case where each spring's model acts within its own amplitude zone, it is possible to represent the total backbone in the form of a summation of several typical backbone curves.

3. DAMPING NONLINEARITIES IN VIBRATION SYSTEMS

The instantaneous damping characteristics of a vibration system are determined from the form of the symmetric frictional force (dissipative function) as a function of the displacement or velocity. When a vibrating system has known nonlinear damping characteristics the corresponding instantaneous damping parameters are also known functions of the amplitude and/or the forced frequency. These dependencies allow us to estimate the type of nonlinearity and the value governing parameters, e.g. the different kinds of damping: dry friction, structural and viscous. Taking into account the analytic signal representation enables one to estimate not only the elastic force but also the damping force characteristics.

Naturally, damping is a complex phenomenon and more than one type of damping may exist in the same real structure. Considering the total frictional force as a sum of typical fictional elements, one can write an expression for the total damping coefficient as a sum of the typical dependencies. However, the decomposition of this total damping coefficient has no simple solution. Only in the case when each simple damping mechanism operates at a different range of amplitude and/or forced frequency we can obtain a unique interpretation of the general damping dependence. The instantaneous damping coefficient can be determined from the form of the symmetric dissipative function. In the particular case of a linear system, the instantaneous natural frequency and the instantaneous damping coefficient or decrement do not vary in time. In the more general case of nonlinear system identification, the instantaneous damping coefficient and the natural frequency become functions of the amplitude and frequency.

If nonlinear dissipative forces are operating in the vibration system, the values obtained from the instantaneous damping coefficient may depend on the instantaneous amplitude. Vibrating systems can sometimes exhibit their damping nonlinearities only in the small- or large-amplitude range, depending on the type of nonlinear damping force that prevails. As an example of a small-amplitude nonlinear behavior we mention a system with Coulomb (dry) friction: a plot of the logarithmic decrement versus vibration amplitude yields as a monotonic hyperbola (Feldman and Braun, 1993). The presence of dry friction together with the viscous friction is modeled by

$$\ddot{x} + 2h_{\mathrm{vis}}\dot{x} + h_{\mathrm{dry}}\mathrm{sgn}(\dot{x}) + \omega_0^2 x = 0. \tag{7}$$

Equation (7) shows that for small vibration amplitudes, the logarithmic decrement increases significantly.

In other cases, such as in nonlinear friction caused by turbulence, the nonlinear damping behavior appears in the large-amplitude range. However, in practice, the small damping forces have practically no effect on the mechanical system's backbone curve in this case.

4. THEORETICAL BASIS FOR NONLINEAR SYSTEM IDENTIFICATION

The time domain techniques based on the Hilbert transform allow a direct extraction of linear and nonlinear system parameters from a measured time signal of input and output of the vibration system. The proposed method of free and forced vibration analysis determines instantaneous modal parameters even when the input signal consists of a high sweep-rate oscillating signal. Such a direct determination of the relationship between the amplitude and natural frequency, which characterizes the elastic properties, and the relationship between the amplitude and damping characteristics enables one to perform an efficient nonlinear system testing procedure without having to conduct long forced response experiments.

4.1. Vibration Analysis Methods

The time domain HT identification methods, namely FREEVIB and FORCEVIB, were proposed as nonparametric methods for identification of instantaneous modal parameters, including determination of system backbone, damping curves, and static force characteristics (Feldman, 1994, 1997). These HT-based methods are suggested for the identification of linear and nonlinear systems under free or forced vibration conditions.

A second-order conservative system with a nonlinear restoring force $k(x) = \omega_0^2(x)$, a nonlinear damping force $h(\dot{x})\dot{x}$, and a solution of the form $x(t) = A(t)\cos[\omega(t)t]$ can be represented by

$$\ddot{x} + h(\dot{x})\dot{x} + \omega_0^2(x) = 0. \tag{8}$$

The FREEVIB method is based on the analytic signal $X(t) = x(t) + j\tilde{x}(t)$, where $\tilde{x}(t)$ is the HT of the solution $x(t)$. The method uses the envelope and phase representation $X(t) = A(t)e^{j\psi(t)}$, where $A(t)$ is the instantaneous envelope (or magnitude), and $\psi(t)$ is the instantaneous phase; both are real functions so $x(t) = A(t)\cos(\psi(t))$, $\tilde{x}(t) = A(t)\sin(\psi(t))$, $A(t) = \sqrt{x^2(t) + \tilde{x}^2(t)}$, and $\psi(t) = \arctan\left(\frac{\tilde{x}(t)}{x(t)}\right)$.

In conclusion we can state that both the envelope and phase are available as functions of time if $x(t)$ is known and $\tilde{x}(t)$ can be computed according to the HT filter. The derivatives can also be computed, either directly or using the relations

$$\dot{X} = X\left(\frac{\dot{A}}{A} + i\dot{\psi}\right), \quad \ddot{X} = X\left(\frac{\ddot{A}}{A} - \dot{\psi}^2 + 2i\dot{A}\psi + i\ddot{\psi}\right), \tag{9}$$

where

$$\dot{\psi}(t) = \omega(t) = \frac{x(t)\dot{\tilde{x}}(t) - \dot{x}(t)\tilde{x}(t)}{A^2(t)} = \mathrm{Im}\left[\frac{\dot{X}(t)}{X(t)}\right]$$

is the instantaneous frequency of the solution $x(t)$.

Now, consider the equation of motion (8), with $h(\dot{x}(t)) = h(t)$and $\omega_0^2(x(t)) = \omega_0^2(t)$ considered purely as functions of time. As the functions $h$and ω_0^2will generally be low-order polynomials of the envelope A, they will have a lowpass characteristic. If the resonant frequency of the system is high, $x(t)$ will, roughly speaking, have a highpass characteristic. This means that $h$and y can be considered as nonoverlapping signals, as can ω_0^2 and x. If the HT of (8) is taken, it will pass through the functions h and ω_0^2. Further, the transform commutes with differentiation, so

$$\ddot{\tilde{x}} + h(\dot{x})\dot{\tilde{x}} + k(\tilde{x}) = 0. \tag{10}$$

Adding (8) and multiplying by j yields a differential equation for the analytic signal X, i.e. $\ddot{X} + h(t)\dot{X} + \omega_0^2(t) = 0$, or the quasi-linear form

$$\ddot{X} + h(A)\dot{X} + \omega_0^2(A) = 0. \tag{11}$$

Now, the derivatives $\dot{X}$ and $\ddot{X}$ are known functions of A and ω by (9). Substituting the derivatives (9) in the equation (11) yields

$$X\left[\frac{\ddot{A}}{A} - \omega^2 + \omega_0^2 + h\frac{\dot{A}}{A} + j\left(2\frac{\dot{A}}{A}\omega + \dot{\omega} + h\omega\right)\right] = 0.$$

Separating out the real and imaginary parts gives

$$\omega_0^2(t) = \omega^2 - \frac{\ddot{A}}{A} + \frac{2\dot{A}^2}{A^2} + \frac{\dot{A}\dot{\omega}}{A\omega}; \quad h_0(t) = -\frac{\dot{A}}{A} - \frac{\dot{\omega}}{2\omega}, \tag{12}$$

and these are the basic equations of the theory.

The inverse of the latter mapping $A(\omega_0)$ is sometimes referred to as the backbone curve of the system. Note that there are no assumptions on the forms of $A(\omega_0)$and $A(h)$, the method is truly nonparametric.

In the first stage of the identification technique, the envelope $A(t)$ and the instantaneous frequency $\omega(t)$ are extracted from the response and excitation signals on the basis of the HT signal processing. In the following stage, by applying the multiplication property of the HT for overlapping functions to the equation of motion, the instantaneous undamped natural frequency and the instantaneous damping coefficient are calculated according to (12), where $A(t)$ and $\omega(t)$ are the envelope and the instantaneous frequency of the measured vibration.

In the last stage of lowpass filtering the set of duplet modal parameters (the instantaneous natural frequency $\langle\omega_0^2(t)\rangle$ and instantaneous damping $\langle h(t)\rangle$) of each natural mode (12) of vibration are defined. As a result of the HT method, the corresponding set of the modal parameters, as functions of the envelope (the skeleton and the damping curves) of each natural mode of vibration, describe the structure's dynamical behavior.

4.2. Computed Harmonic Response

It is convenient to present the amplitude-dependent response with the newly identified modal parameters in the form of the amplitude response to harmonic excitation. The standard excitation of sinusoidal force is applied to the input of the system at every frequency in the specified range. Thus, the amplitude of the tested system's harmonic response expressed as a SDOF can be written as

$$A = \frac{2A_{\max}h(A)}{\omega_0(A)\sqrt{\left[1-\omega^2/\omega_0^2(A)\right]^2 + 4h^2(A)\omega^2/\omega_0^2(A)}}. \tag{13}$$

Here A is the steady-state vibration amplitude (proportional to the magnitude of the frequency response function), $A_{\max}$ is the maximum value of the vibration amplitude, $\omega = 2\pi f$ is the angular frequency of vibration, $\omega_0(A) = 2\pi f_0(A)$ is the angular natural undamped frequency as a function of amplitude, and $h(A)$ is the damping coefficient, as a function of the amplitude. For sake of plotting the estimated response of the tested system using the identified modal parameters, the last equation should be further inverted

$$\omega^2 = \omega_0^2(A) - 2h^2(A) \pm 2\omega_0(A)h(A)\sqrt{\frac{A_{\max}^2}{A^2} - 1 + \frac{h^2(A)}{\omega_0^2(A)}},$$

$$0 \le A \le A_{\max}\left[1 - \frac{h^2(A)}{\omega_0^2(A)}\right]^{-\frac{1}{2}}. \tag{14}$$

Using Equation (14) we can plot the tested system's harmonic response amplitude in a common form as a separate resonance curve together with the system's backbone skeleton curve.

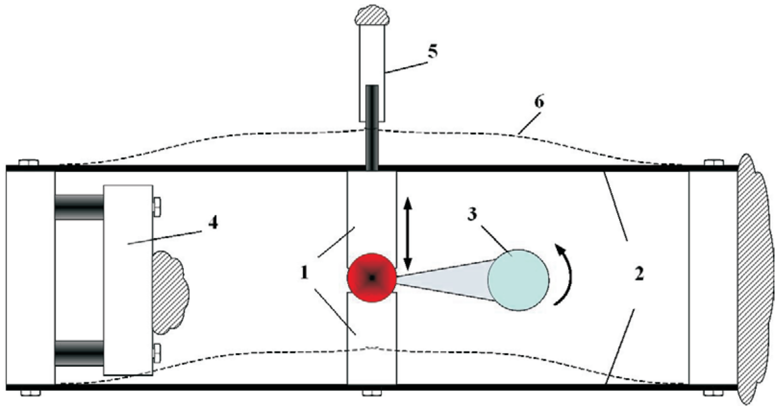
4.3. Interpretation of the Static Force Characteristics

The HT identification, being a nonparametric method, forms the resultant nonlinear elastic and damping force characteristics by direct extraction of the skeleton and the damping curves. In this final stage, the symmetric nonlinear elastic and the damping force characteristics are estimated according to Equation (12):

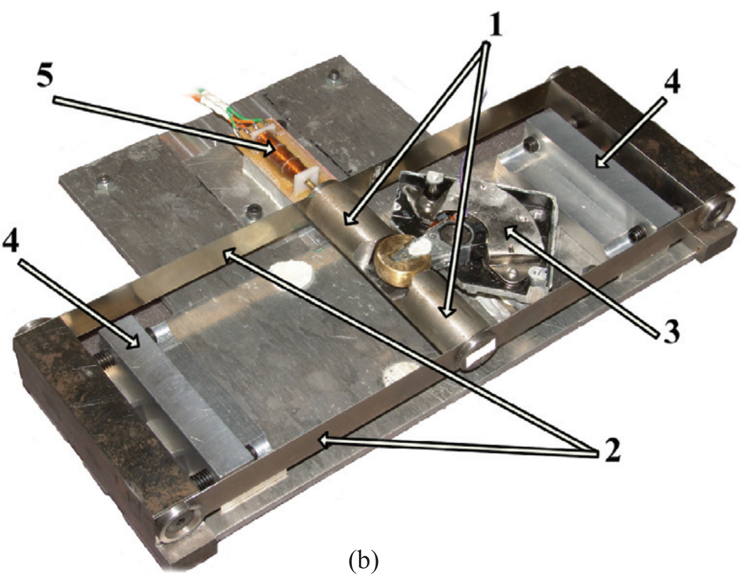
$$k(x) \approx \begin{cases} \left\langle \omega_0^2(t) \right\rangle A(t), & x > 0 \\ -\left\langle \omega_0^2(t) \right\rangle A(t), & x < 0 \end{cases}; \quad h(\dot{x})\dot{x} \approx \begin{cases} \langle h_0(t) \rangle\, a_{\dot{x}}(t), & \dot{x} > 0 \\ -\langle h_0(t) \rangle\, a_{\dot{x}}(t), & \dot{x} < 0 \end{cases} \tag{15}$$

where $A(t)$ and $a_{\dot{x}}(t)$ are the envelope of the displacement and the velocity of the vibration motion, respectively. The nonlinear spring force function that was identified from measured vibrations of the SDOF system will correspond to the system's initial static elastic force characteristics per unit mass.

In general, even a SDOF nonlinear system could include several elastic and damping elements, and these elements are combined integrally through parallel and/or series connections. In such complicated cases of system identification, there is no unique solution for



(a)



(b)

Figure 2. The experimental stand: (a) mass, (2) elastic beams (ruler springs), (3) actuator, (4) tension element, (5) LVDT position sensor, (6) deflected position.

the decomposition of the resultant force characteristics, and one should use some additional information of the model structure and its element combination.

5. THE STRUCTURE UNDER TEST

An experimental vibrating structure was constructed with the following features: the structure consisted of a mass attached to a heavy base by means of two springs, as shown in Figure 2. The mass between the springs can move horizontally about its equilibrium position while the springs vibrate as fixed-end beams. The test rig included a special pre-tensioning mechanism, coupled to a base plate. The experimental vibrating system included an external actuator allowing for the application of variable force excitation and a linear variable differential transformer (LVDT) sensor to measure the mass motion. The actuator consists of a voice coil actuator and a mechanism that converts angular motion into linear motion. The actuator contains some backlash while the elastic beams exhibit pre-tensioning and stiffening effects.

6. EXPERIMENTAL RESULTS AND DISCUSSION

In the present work, the identification was carried out on the basis of experimentally determined instantaneous characteristics of free and forced vibration response signals measured from the test stand. Applying the nonparametric HT identification technique and the instantaneous signal frequency estimation, one can compute the signal's envelope and the produce the structural parameters. These steps do not pose severe computational or procedural difficulties and can be performed in a short time for an arbitrary type of nonlinearity inherent to the system.

6.1. Free Vibration Identification

The free vibration displacement signal was produced by abruptly stopping a forced excitation that was exciting the system at resonance (here, 13 Hz). The actuator under free vibration generated a repeated excitation sequence burst consisting of periodic excitation (at 13 Hz) that stopped for a short while and then restarted. Figure 3 shows an example of four repeated patterns of the measured displacement. Naturally, the free vibration decay corresponds only to the decaying part of each pattern that takes place when the actuator is switched off. The experimental investigation shows that the tested structure can be represented only by a SDOF system.

The backbones obtained according to Equation (12) of four repeated impulses are shown concurrently in Figure 4 (dashdot line) together with the corresponding amplitude response according to Equation (14). All of these backbones practically coincide and indicate that the natural frequency is an amplitude-dependent function. The skeleton curve tips out of the vertical to the right for both the small (less than 3×10^{-3} m) and large amplitudes (more than 5×10^{-3} m). This means that the tested structure includes two different types of nonlinear stiffness element: the preloaded amplitudes and the hardening spring. For the large amplitudes and a hardening spring, a polynomial curve fit (Equation (6)) of the estimated nonlinear skeleton curve gives the following form $\omega^2(A) = 4\pi^2 12.67^2 + \frac{3}{4}0.46A^2$ [rad/s]^2. For the small amplitudes from the same skeleton curve according to Equation (1) a very small preloading force value equal to $F_0/k = 1.2 \cdot 10^{-5}$ [m] was estimated.

The obtained damping curves for the four excitation patterns practically coincide showing little variation in the estimated average damping coefficient $h = 2.5$ [s^{-1}] (Figure 5a). Estimation of the damping allows us to construct a curve showing the amplitude of the damped response for the free vibration regime (Figure 4, dashed line) and the damping static force characteristics (Figure 5b). As can be seen, the damping force characteristics indicate

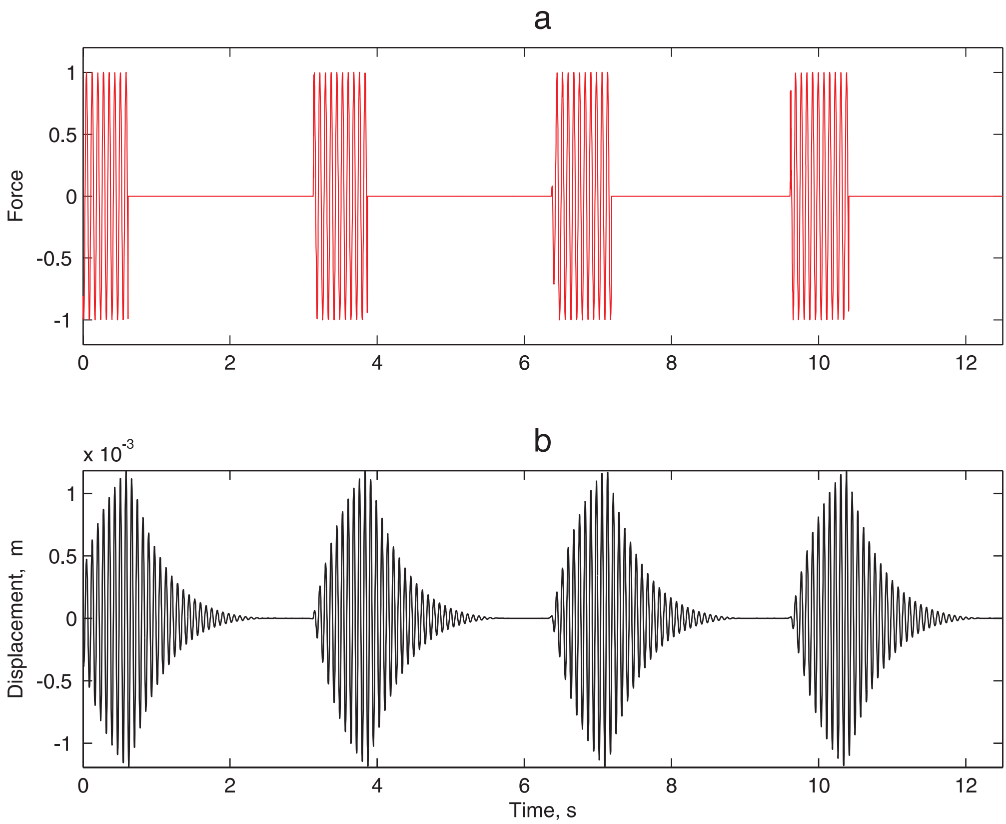


Figure 3. The measured time histories: (a) the repeated force excitation; (b) the output displacement.

that a combination of the linear viscous friction and the dry friction (Equation (7)) exists. The dry friction force for unit mass is obtained after performing a polynomial curve-fitting and it is equal to 0.015 [m/s^2].

The resultant identified model for unit mass takes the form of a SDOF vibration system

$$\ddot{x} + 5\dot{x} + 0.015\mathrm{sgn}(\dot{x}) + (2\pi\, 12.67)^2\, x + 0.46x^3 + 1.2 \times 10^{-5}\,(2\pi\, 12.67)^2\,\mathrm{sgn}(x) = 0. \quad (16)$$

For the range $0 < A_x < 1.5 \times 10^{-3}$ [m], it is clear that the nonlinear spring and the nonlinear damping cannot be ignored in both the large- and small-amplitude range.

The results of the HT identification of free vibration (Equation (16)) describe the main system's linear and nonlinear properties including the skeleton and damping curves, as well as the relative static stiffness and damping force characteristics per unit mass. These results constitute the basis for the identification of the model, but free vibration analysis does not restore the absolute system mass and stiffness values. To estimate the absolute values of the system parameters we provide a forced vibration regime of the system.

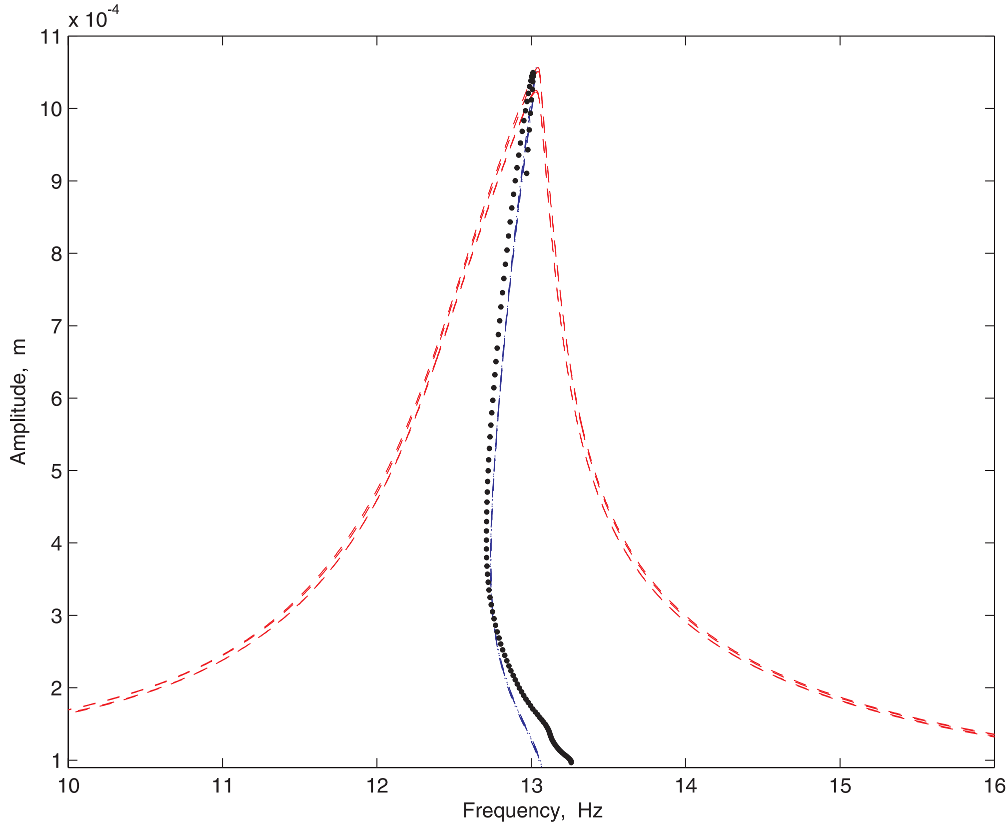


Figure 4. The experimental skeleton curves and frequency response: the skeleton curves of free vibrations (- · · -); the frequency response functions (- -); the skeleton curves of forced vibrations (· · ·).

6.2. Forced Vibration Identification

During this test, the forced vibrations were produced by the actuator (exciter) with a continuous frequency sweep in the range of 5–20 Hz. The force generated by the actuator is shown in Figure 6a, and the corresponding forced vibration is shown in Figure 6b. The HT identification according to (Feldman, 1994) uses these input and output time histories, where the displacement and the force are presented in the time domain.

The obtained results as well as the results from the free vibration identification evidently include the same skeleton and damping curves, and also the same static stiffness and damping force characteristics. For example, Figure 4 (dotted line) shows the skeleton curve, which practically coincides with the same curve from the free vibration regime. However, the forced vibration identification is able to restore the absolute value of the reduced mass and the absolute value of the stiffness. Thus, the obtained absolute mass value of all of the moving parts of the experimental stand is equal to 0.27 kg. The obtained mass and the natural frequency give the value of the reduced average static stiffness value of the structure: $k = m\omega_0^2 = 0.27\,(2\pi \cdot 12.7)^2 \approx 1.7 \cdot 10^3$ [N/m].

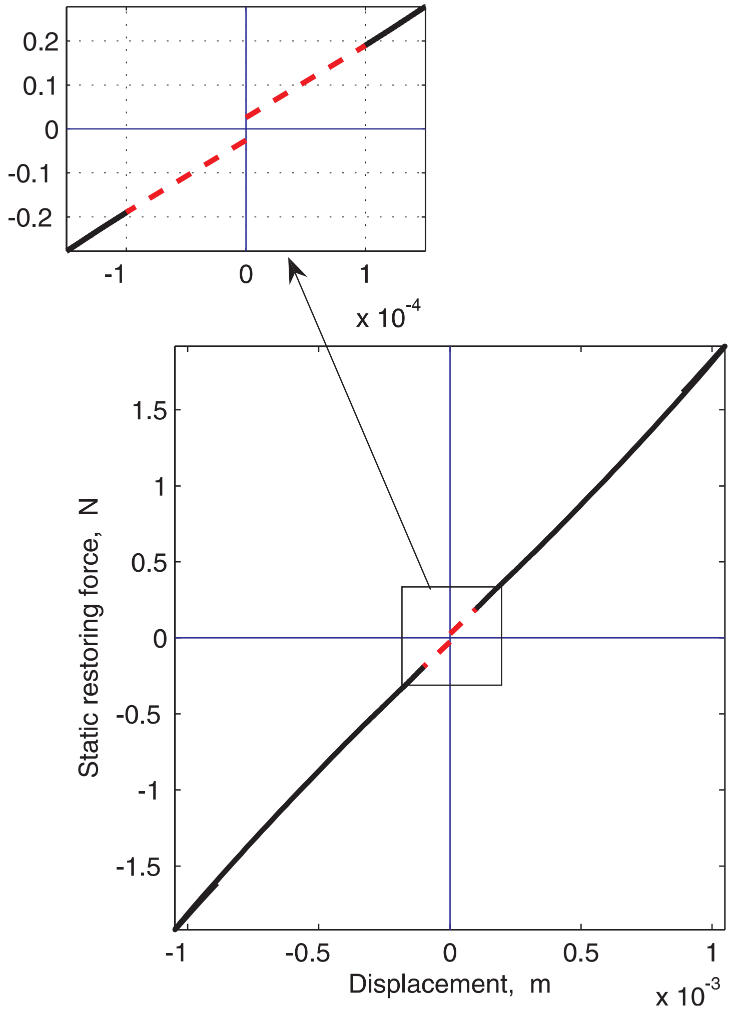


Figure 5. The experimental stiffness static force characteristics.

The obtained mass value also determines the both absolute (static) force characteristics (the stiffness and damping, Figure 7b) and the final model of forced vibration of the system:

$$0.27\left[\ddot{x} + 5\dot{x} + 0.015\mathrm{sgn}(\dot{x}) + (2\pi\, 12.67)^2\, x + 0.46x^3 + \frac{0.02}{0.27}\,\mathrm{sgn}(x)\right] = F(t), \qquad (17)$$

$0 < A_x < 1.5 \times 10^{-3}$ [m], where 0.27 is the estimated mass value, $F(t)$ is the external force. The last formula practically repeats Equation (16), but now the forced vibration model incorporates the identified mass value. The identified model, having nonlinear elastic and damping forces, describes the system's motion under different types of input excitation.

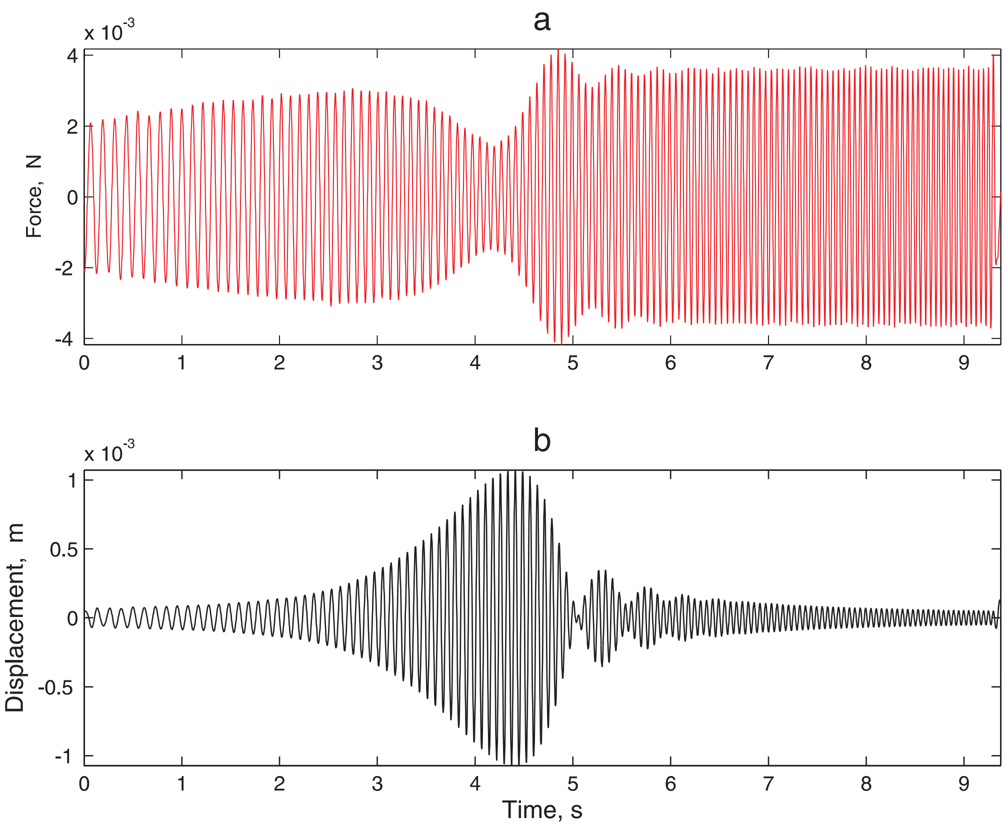


Figure 6. The measured time histories: (a) the input sweeping force excitation; (b) the output displacement.

7. CONCLUSIONS

In this paper we have presented the results obtained by the HT nonparametric identification of a real mechanical system consisting of a mass, spring and damping. The input force and the system's displacement response were measured under free vibrations (transient) and also under harmonic force excitation.The identification was carried out by means of the HT technique of signal processing for nonlinear systems. This technique is based on the analysis of the input and output signals of a system: its envelope and its instantaneous frequency in the time domain.

The HT-based technique enables us to estimate directly the system instantaneous dynamic parameters (i.e. natural frequencies, damping characteristics) and also their dependence on the vibration amplitude and frequency.

This direct time domain techniques allows for a direct extraction of the linear and nonlinear system parameters from the measured time signals of input and output.

The model of the tested structure was created by curve fitting the experimentally obtained skeleton and damping curves by the theoretical nonlinear functions. The obtained

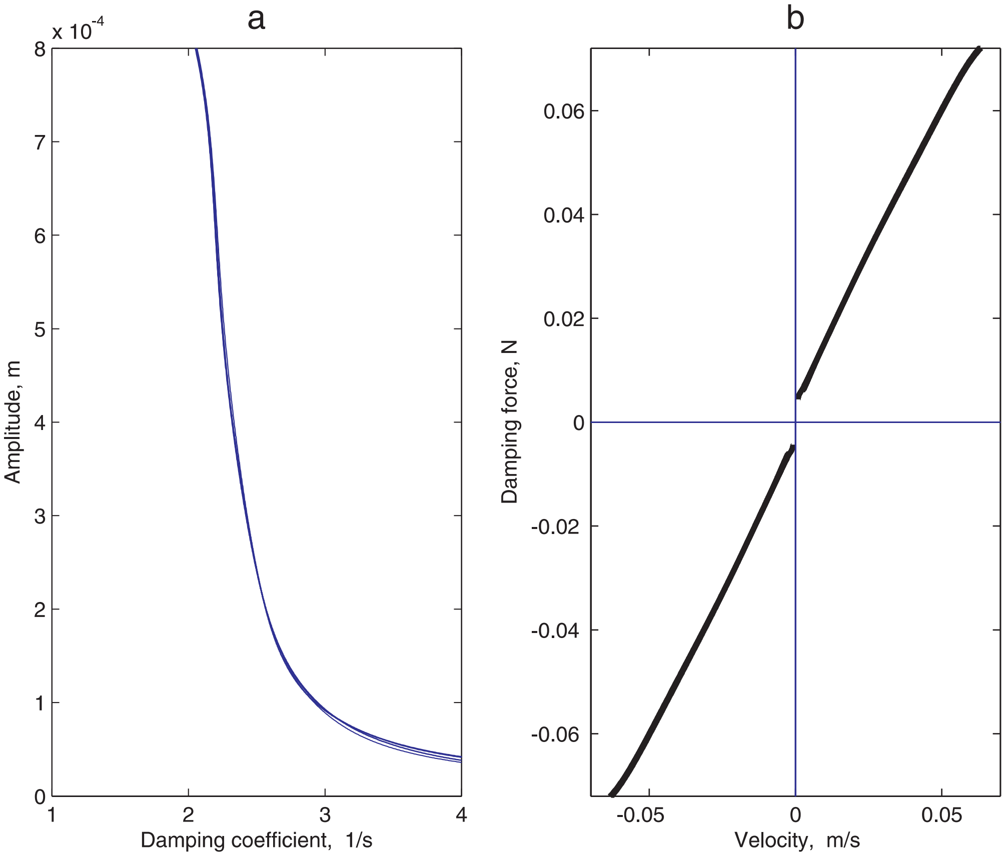


Figure 7. (a) The experimental damping curve and (b) the friction static force characteristics.

results may be used to verify and validate the model under different conditions, to simulate possible solutions generated by any other input force and to find a control scheme that provides a desired vibration response.

The introduced identification method of free and forced vibration analysis, which determines instantaneous modal parameters, contributes to efficient and more accurate testing of nonlinear oscillatory systems, avoiding time-consuming measurement and analysis.

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